

Color-singlet J/ψ production at $\mathcal{O}(\alpha_s^6)$ in Υ decay

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To clarify the conflict between the theoretical predictions and experimental measurements of the inclusive J/ψ production in Υ decay, We consider the α_s^6 order color-singlet contributions of processes $\Upsilon \rightarrow J/\psi + gg$ and $\Upsilon \rightarrow J/\psi + gggg$. Both the branching ratio and J/ψ momentum spectrum are calculated, and the branching ratio (4.7×10^{-4}) is larger than the leading-order contribution ($\alpha_s^5, \Upsilon \rightarrow J/\psi + c\bar{c}g$). Together with the QCD and QED leading-order contributions considered in our previous work, the color-singlet prediction of the branching ratio for the direct J/ψ production is $\text{Br}(\Upsilon \rightarrow J/\psi_{\text{direct}} + X) = 0.90_{-0.31}^{+0.49} \times 10^{-4}$, which is still about 3.8 times less than the CLEO measurement. We also obtain a preliminary color-singlet prediction of $R_{cc} = \frac{\mathcal{B}(\Upsilon \rightarrow J/\psi + c\bar{c} + X)}{\mathcal{B}(\Upsilon \rightarrow J/\psi + X)}$ and find the value $0.39_{-0.20}^{+0.21}$ is much larger than the color-octet predictions, and suggest to measure this quality in future experimental analysis.

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The existence of a hierarchy of energy scales: $m_Q \gg m_Q v \gg m_Q v^2$ makes the heavy quarkonium system be an ideal laboratory to study both perturbative and non-perturbative aspects of QCD, where v , being assumed to be much smaller than 1, is the velocity of heavy quark in the rest frame of the heavy meson. And it is commonly believed that the nonrelativistic QCD (NRQCD)[1] effective theory provides a rigorous factorization formalism to separate the physics in different scales. The NRQCD not only covers the results of previous color-singlet(CS) model predictions, where at short distance the $Q\bar{Q}$ can only be in the color-singlet configuration with the same quantum numbers as the corresponding heavy quarkonium states, it also includes the contribution of $Q\bar{Q}$ in the color-octet(CO) configuration at short distance.

Despite of the impressive success of the NRQCD, the role of the CO is not well established yet, particularly in J/ψ production case. The substantial theoretical progress in the next-to-leading-order(NLO) calculations shows that there is not a convincing mechanism to explain the J/ψ production data in varies experiments self-consistently yet. For $J/\psi + c\bar{c} + X$ [2] and $J/\psi + X_{\text{non-}c\bar{c}}$ [3] production in e^+e^- annihilation at B-factories, the CS processes themselves can account for the cross sections when the NLO QCD corrections[4–7] and relativistic corrections [8, 9] are taken into account, which leaves very little room for the CO contribution[10]. For J/ψ production from Z decay, recent NLO QCD correction calculation in the CS [11] gives just one-half of the experimental measurement and the CO contribution might be able to explain the other-half. The transverse momentum p_t distribution of J/ψ photoproduction and polar-

ization parameters at HERA can not be well described by the CS[12, 13] at QCD NLO, it seems that p_t distribution of J/ψ photoproduction can be explained by the CO and CS contribution together at QCD NLO[15]; For J/ψ hadroproduction, together with the CS [16–18] contribution and the CO contribution [19, 20] at NLO in α_s , we can not describe the Tevatron results about the p_t distribution of J/ψ production and polarization simultaneously yet.

In order to clarify such a puzzling theoretical situation, it is worth to further investigate some other J/ψ production processes, one of which is the inclusive J/ψ production in Υ decay. From the theoretical point of view, because Υ predominately decays into three gluons via $b\bar{b}$ annihilation, it is proposed[21, 22] that in the rich-gluon final state environment abundant J/ψ can be produced through $c\bar{c}$ pair in CO 3S_1 configuration. Hence, the inclusive J/ψ production in Υ decay will be an another good probe to discriminate the CS and CO mechanism. And the present CO predictions of the branching ratio is $\mathcal{B}(\Upsilon \rightarrow J/\psi + X) = 6.2 \times 10^{-4}$ [21, 22] with about 10% feed-down contribution from $\psi(2S)$ and 10% from χ_{cJ} [24]. While the correct CS result, which was overestimated by about an order in magnitude before [25, 26], is only 4.2×10^{-5} [27]. On the experimental side, the branching ratio for $\Upsilon \rightarrow J/\psi + X$ has been measured by a few collaborations about twenty years ago [28–30], and recently, the more precise measurement carried by the CLEO Collaboration gave[31]

$$\mathcal{B}(\Upsilon \rightarrow J/\psi + X) = (6.4 \pm 0.4 \pm 0.6) \times 10^{-4}. \quad (1)$$

It can be seen that the CLEO result is in good agreement with the CO prediction, but the J/ψ momentum distribution measured by CLEO [31] is much softer than the CO predictions [21, 22]. In a very recent work[23],

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it is found that the momentum spectrum can be significantly softened after combining the NRQCD and Soft Collinear Effective Theory (SECT) in the kinematic end-point region. However, it yields a much smaller branching ratio. This may indicate that the CO processes do not contribute dominantly.

In the CS case, the results at QCD and QED leading-order (LO) [27] is about an order of magnitude smaller than the experimental result. To investigate the CS contributions for the J/ψ production in Υ decay more precisely, we consider the $\mathcal{O}(\alpha_s^6)$ contributions through $\Upsilon \rightarrow J/\psi + gg$ and $\Upsilon \rightarrow J/\psi + gggg$ processes. It was refereed in Ref. [24] that this two processes had been crudely estimated and the branching ratio is a few of $\times 10^{-4}$, which is comparable to CLEO data. Since the former calculation refereed is rough without any publication and detail, it is necessary to give an exact result and a complete analysis of these two processes. To perform the calculations, we employ the Feynman Diagram Calculation (FDC) package [32].

According to NRQCD factorization approach, at the leading order of v_b^2 and v_c^2 , the CS contribution to $\Upsilon \rightarrow J/\psi + X$ is expressed as:

$$d\Gamma(\Upsilon \rightarrow J/\psi + X) = d\hat{\Gamma}(b\bar{b}[^3S_1, \underline{1}] \rightarrow c\bar{c}[^3S_1, \underline{1}] + X) \times \langle \Upsilon | \mathcal{O}_1(^3S_1) | \Upsilon \rangle \langle \mathcal{O}_1^\psi(^3S_1) \rangle, \quad (2)$$

where $d\hat{\Gamma}$ is partonic partial decay width which can be calculated perturbatively. By dimension analysis, it is easy to derive out that the general expression of the partial width is written as

$$\hat{\Gamma} = \frac{1}{3(2N_c)^2} \frac{\alpha_s^6}{m_b^5} f(r) \quad (3)$$

where $r = m_c/m_b$ is a dimensionless parameter and f is a process dependent function of r . The $\langle \Upsilon | \mathcal{O}_1(^3S_1) | \Upsilon \rangle$ and $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$ in Eq. [2] are the nonperturbative matrix elements, which will be determined phenomenologically. To be consistent with our former work [27], we keep the factor $\frac{1}{3(2N_c)^2}$ explicitly.

The $\mathcal{O}(\alpha_s^6)$ $\Upsilon \rightarrow J/\psi + gg$ process is of 36 one-loop Feynman Diagrams at leading-order. To simplify the cal-

culational, each diagram is summed up with its possible partner diagram which is obtained by reversing the direction of b-quark or c-quark line in fermion loop. Then the diagrams are divided into ten groups, and the representative ones are shown in Fig. [1] and others can be obtained by exchanging the positions of the two final-state gluons. For diagrams in the same group, the amplitude equals to each other when ignoring color factor, thus only the $d^{abc}d^{abc}$ piece in color factor will survive after all the diagrams being summed up in each group. It is found that there are infrared divergences in the amplitudes of diagrams in A1, A2, A4, A5 groups, at least in Feynman gauge. And the divergence terms in A1(A2) group cancel those in A4(A5), then the total amplitude is finite. The amplitude of each diagram in group A3 is finite individually because in such diagrams there are no such a virtual

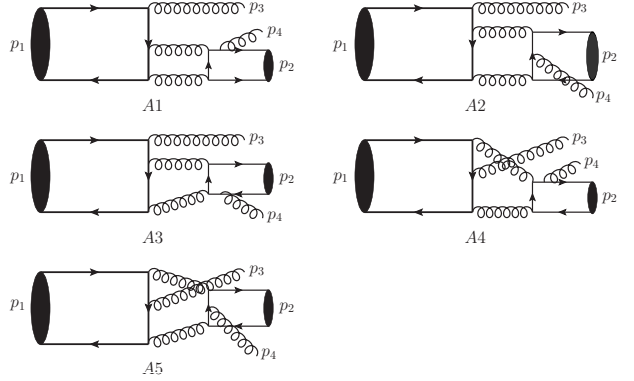


FIG. 1: The typical Feynman diagrams of five of ten groups in the CS $\Upsilon \rightarrow J/\psi + gg$ process, and the other five typical ones can be obtained by exchanging the positions of the two final-state gluons.

gluon which joints two on shell (anti)quarks or one on shell quark and one on shell antiquark.

Before showing the result, we would like to address some non-trivial techniques treatment in the calculation. By applying the FDC package, the general expression of the Feynman amplitude for $\Upsilon(p_1, \epsilon_1) \rightarrow J/\psi(p_2, \epsilon_2) + g(p_3, \epsilon_3) + g(p_4, \epsilon_4)$ process is generated as:

$$\begin{aligned} M = & \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 c_{41} + \epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4 c_{42} + \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3 c_{43} + p_4 \cdot \epsilon_2 p_4 \cdot \epsilon_3 \epsilon_1 \cdot \epsilon_4 c_{19} + p_4 \cdot \epsilon_1 (p_4 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 c_{37} \\ & + p_4 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4 c_{39}) + p_3 \cdot \epsilon_4 [p_4 \cdot \epsilon_1 (\epsilon_2 \cdot \epsilon_3 c_{21} + p_4 \cdot \epsilon_2 p_4 \cdot \epsilon_3 c_9) + p_4 \cdot \epsilon_2 \epsilon_1 \cdot \epsilon_3 c_{29} + p_4 \cdot \epsilon_3 \epsilon_1 \cdot \epsilon_2 c_{31}] \\ & + p_3 \cdot \epsilon_2 [p_4 \cdot \epsilon_3 \epsilon_1 \cdot \epsilon_4 c_{20} + p_4 \cdot \epsilon_1 \epsilon_3 \cdot \epsilon_4 c_{35} + p_3 \cdot \epsilon_4 (\epsilon_1 \cdot \epsilon_3 c_{27} + p_4 \cdot \epsilon_1 p_4 \cdot \epsilon_3 c_{10})] + p_3 \cdot \epsilon_1 [p_4 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 c_{38} \\ & + p_4 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4 c_{40} + p_3 \cdot \epsilon_4 (\epsilon_2 \cdot \epsilon_3 c_{22} + p_4 \cdot \epsilon_2 p_4 \cdot \epsilon_3 c_{11}) + p_3 \cdot \epsilon_2 (\epsilon_3 \cdot \epsilon_4 c_{36} + p_3 \cdot \epsilon_4 p_4 \cdot \epsilon_3 c_{12})] \\ & + p_2 \cdot \epsilon_3 \{ p_4 \cdot \epsilon_2 \epsilon_1 \cdot \epsilon_4 c_{17} + p_4 \cdot \epsilon_1 \epsilon_2 \cdot \epsilon_4 c_{33} + p_2 \cdot \epsilon_4 [\epsilon_1 \cdot \epsilon_2 c_{26} + p_3 \cdot \epsilon_2 p_4 \cdot \epsilon_1 c_6 + p_3 \cdot \epsilon_1 (p_3 \cdot \epsilon_2 c_8 \\ & + p_4 \cdot \epsilon_2 c_7) + p_4 \cdot \epsilon_1 p_4 \cdot \epsilon_2 c_5] + p_3 \cdot \epsilon_4 (\epsilon_1 \cdot \epsilon_2 c_{25} + p_4 \cdot \epsilon_1 p_4 \cdot \epsilon_2 c_1) + p_3 \cdot \epsilon_2 (\epsilon_1 \cdot \epsilon_4 c_{18} + p_3 \cdot \epsilon_4 p_4 \cdot \epsilon_1 c_2) \\ & + p_3 \cdot \epsilon_1 (\epsilon_2 \cdot \epsilon_4 c_{34} + p_3 \cdot \epsilon_4 p_4 \cdot \epsilon_2 c_3 + p_3 \cdot \epsilon_2 p_3 \cdot \epsilon_4 c_4) \} + p_2 \cdot \epsilon_4 [p_3 \cdot \epsilon_2 (\epsilon_1 \cdot \epsilon_3 c_{28} + p_4 \cdot \epsilon_1 p_4 \cdot \epsilon_3 c_{14}) \\ & + p_3 \cdot \epsilon_1 (\epsilon_2 \cdot \epsilon_3 c_{24} + p_3 \cdot \epsilon_2 p_4 \cdot \epsilon_3 c_{16} + p_4 \cdot \epsilon_2 p_4 \cdot \epsilon_3 c_{15}) + p_4 \cdot \epsilon_1 (\epsilon_2 \cdot \epsilon_3 c_{23} + p_4 \cdot \epsilon_2 p_4 \cdot \epsilon_3 c_{13}) \\ & + p_4 \cdot \epsilon_2 \epsilon_1 \cdot \epsilon_3 c_{30} + p_4 \cdot \epsilon_3 \epsilon_1 \cdot \epsilon_2 c_{32}] \end{aligned} \quad (4)$$

where c_i , $i = 1, \dots, 43$ are the coefficients of the Lorentz

structure and all the loop diagrams contribute to their

values. From the general expression in Eq.(4), it is easy to understand why the tensor reduction procedures are very complicated and will generate complicated results for the coefficients c_i .

These complicated results of c_i may contain fake pole structures and cause big number cancellation problem, finally will spoil numerical calculation in limited precision case. In fact, it really took place in our numerical calculation. We found it is impossible to control the cancellation of the big numbers and to obtain correct results in double precision FORTRAN calculation. In quadruple precision FORTRAN calculation, we can obtain the correct results by introducing cut-conditions to control these fake poles in phase space integration. To demonstrate that our treatment is suitable, we define a cut condition parameter in phase space integration as

$$A = \frac{\Gamma(|M|^2 = 1, \text{ with cuts})}{\Gamma(|M|^2 = 1, \text{ without cuts})} \quad (5)$$

and calculate the partial decay width numerically with different cut condition parameter. The results are:

$$\begin{aligned} \Gamma &= a(1.440 \pm 0.001) \times 10^{-10} \text{ for } A = 1 - 0.04 \\ \Gamma &= a(1.446 \pm 0.001) \times 10^{-10} \text{ for } A = 1 - 0.005 \\ \Gamma &= a(1.450 \pm 0.001) \times 10^{-10} \text{ for } A = 1 - 0.0005, \end{aligned} \quad (6)$$

where all the calculations are under control in quadruple precision FORTRAN calculation, and a is just a constant number. We have tried to do the calculation with $A = 1 - 0.00005$ and found a divergent result. It means that the big number cancellation will lose control even in quadruple precision FORTRAN when A is too close to 1. Therefore, our following calculations for $\Upsilon \rightarrow J/\psi + gg$ are based on the cut condition parameter $A = 1 - 0.0005$. Moreover, to check the gauge invariance, in the expression of Eq.(4) we replace the gluon polarization vector ϵ_3 (or ϵ_4) by its 4-momentum p_3 (or p_4) in the final numerical calculation. Definitely the result must be zero and our results reproduce it.

Now, we proceed to present our results. Since the two interior gluons can be on shell simultaneously in this process, the amplitude will be complex-valued. And we use the superscript ‘Im’ to denote the contribution of the real process $\Upsilon \rightarrow 3g$ followed by $gg \rightarrow J/\psi + g$. Setting $m_c = m_{J/\psi}/2 = 1.548 \text{ GeV}$ and $m_b = m_\Upsilon/2 = 4.73 \text{ GeV}$, which corresponds to $r = 1.548/4.73 = 0.327$, we get $f_{gg}(r) = 2.07$ and $f_{gg}^{\text{Im}}(r) = 0.741$ which is about 1/3 of the total. To show the dependence of f on r , we also list some of the numerical results of $f(r)$ in Tab.I, where r is in the range of $0.275 < r < 0.381$, which is obtained by fixing the value of m_b and varying m_c from 1.3GeV to 1.8GeV GeV[25]. For comparison we also list the results of $f_{ccg}(r)$ for the $\mathcal{O}(\alpha_s^5) \Upsilon \rightarrow J/\psi + c\bar{c} + g$ process. It can be seen that both the value of $f_{gg}(r)$ and that of $f_{gg}^{\text{Im}}(r)$ do not change sharply, when r goes from 0.275 to 0.381, and this behavior is quite different from what happens to $f_{ccg}(r)$. Also the ratio of $f_{gg}^{\text{Im}}(r)$ to $f_{gg}(r)$ changes very little with r .

TABLE I: The values of $f(r)$ for $J/\psi + c\bar{c} + g$ (f_{ccg}), $J/\psi + gg$ (f_{gg} and f_{gg}^{Im}), $J/\psi + gggg$ (f_{4g}) production in Υ decay with different inputs of $r = \frac{m_c}{m_b}$.

r	$f_{ccg}(r)$	$f_{gg}(r)$	$f_{gg}^{\text{Im}}(r)$	$f_{4g}(r)$
0.275	0.904	2.94	1.02	1.35×10^{-2}
0.296	0.567	2.54	0.892	1.08×10^{-2}
0.317	0.345	2.21	0.786	0.880×10^{-2}
0.327	0.269	2.07	0.741	0.800×10^{-2}
0.338	0.202	1.94	0.696	0.721×10^{-2}
0.361	0.105	1.68	0.612	0.585×10^{-2}
0.381	0.055	1.49	0.547	0.490×10^{-2}

There are 216 Feynman diagrams in the CS $\Upsilon \rightarrow J/\psi + gggg$ process, and the typical one is shown in Fig.2. It is a tree process without infrared divergence, and the numerical results are calculated straightforwardly with the help of FDC package. When $r = 0.327$, we get $f_{4g}(r) = 0.8 \times 10^{-2}$, which is more than two orders less than $f_{gg}(0.327)$. Some other numerical results of $f_{4g}(r)$ for $0.275 < r < 0.381$ are also listed in Tab.I. Like $f_{gg}(r)$, the function $f_{4g}(r)$ also does not dependent on r seriously, but its value is too small comparing to the values of the f functions of the other processes. One possible reason is that the five-body phase space is much smaller than the three-body phase space.

Besides $f_{gg}(r)(f_{4g}(r))$, the partial decay width $\Gamma(\Upsilon \rightarrow J/\psi + gg(4g))$ also depends on the choice of the values of the two NRQCD long-distance matrix elements, the coupling constant α_s and the b-quark mass m_b . The value of $\langle \mathcal{O}_1^\psi(^3S_1) \rangle \simeq 3 \langle J/\psi | \mathcal{O}(^3S_1) | J/\psi \rangle$ can be extracted from J/ψ decay into e^+e^- by using the upto α_s order result

$$\Gamma(J/\psi \rightarrow e^+e^-) = \frac{2\pi e_c^2 \alpha^2}{3m_c^2} \left(1 - \frac{16\alpha_s}{3\pi}\right) \langle \psi | \mathcal{O}_1(^3S_1) | \psi \rangle. \quad (7)$$

Using $\alpha = 1/128$, $m_c = 1.548 \text{ GeV}$, $\alpha_s(2m_c) = 0.26$, $\Gamma(J/\psi \rightarrow e^+e^-) = 5.54 \text{ keV}$ [33], we get $\langle \mathcal{O}_1^\psi(^3S_1) \rangle = 1.25 \text{ GeV}^3$. And $\langle \Upsilon | \mathcal{O}(^3S_1) | \Upsilon \rangle = 2.92 \text{ GeV}$ is determined in a similar way with $m_b = 4.73 \text{ GeV}$, $\alpha_s(2m_b) = 0.18$, $\Gamma(\Upsilon \rightarrow e^+e^-) = 1.29 \text{ keV}$ [33]. The uncertainty from the choice of the renormalization scale is quite large since there are two typical energy scales m_b and m_c in the calculation. By choosing $\mu = 2m_c$, we find

$$\begin{aligned} \Gamma^g &= \Gamma(\Upsilon \rightarrow J/\psi + gg) + \Gamma(\Upsilon \rightarrow J/\psi + 4g) \\ &= 9.1 \times 10^{-3} \text{ keV} \end{aligned} \quad (8)$$

It corresponds to

$$\begin{aligned} \mathcal{B}^g &= \mathcal{B}(\Upsilon \rightarrow J/\psi + gg) + \mathcal{B}(\Upsilon \rightarrow J/\psi + 4g) \\ &= 1.7 \times 10^{-4} \end{aligned} \quad (9)$$

which is coincident with the rough result mentioned in Ref.[24]. However, the branching ratio becomes much smaller and is only 2.32×10^{-5} when choosing $\mu = 2m_b$.

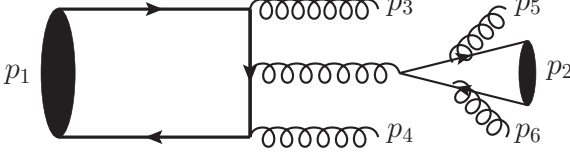


FIG. 2: One of the 216 Feynman diagrams for the CS $\Upsilon \rightarrow J/\psi + ggg$ process.

This is because the processes are at α_s^6 order. Note that, seemingly, the branching ratio also strongly dependent on m_b as m_b^{-5} , but in fact the dependence is m_b^{-3} because of the dependence of the nonperturbative matrix element $\langle \Upsilon | \mathcal{O}^3 S_1 | \Upsilon \rangle$ on m_b from its phenomenological determination. To obtain the above numerical results for the branching ratio, the experimental measurement on the Υ total decay width $\Gamma_\Upsilon = 53\text{keV}$ [33] is used.

Since the J/ψ is produced through the three-gluon decay channel of Υ in these two processes, It is natural to normalize the partial width to the decay width of $\Upsilon \rightarrow ggg$, which at LO in α_s is given by

$$\Gamma(\Upsilon \rightarrow ggg) = \frac{20\alpha_s^3}{243m_b^2}(\pi^2 - 9)\langle \Upsilon | \mathcal{O}^3 S_1 | \Upsilon \rangle. \quad (10)$$

Then branching ratio is expressed in an alternate form

$$\mathcal{B}^g = \Gamma_{\text{Nor}}^g \times \mathcal{B}(\Upsilon \rightarrow ggg), \quad (11)$$

where

$$\Gamma_{\text{Nor}}^g = \frac{81(f_{gg}(r) + f_{4g}(r))\alpha_s^3 \langle \mathcal{O}_1^\psi(^3S_1) \rangle}{20(2N_c)^2 m_b^3 (\pi^2 - 9)}, \quad (12)$$

and $\mathcal{B}(\Upsilon \rightarrow ggg) = 84\%$ is obtained by assuming $\mathcal{B}(\Upsilon \rightarrow ggg) \approx \mathcal{B}(\Upsilon \rightarrow \text{light hadron (LH)})^a$ [33]. To calculate the branching ratio in this way is equivalent to determine $\alpha_s^3 \langle \Upsilon | \mathcal{O}^3 S_1 | \Upsilon \rangle$ from LH decay of Υ and can reduce the uncertainties from α_s . And our following results are all calculated based on Eq.[11].

The numerical results of “f(r)” for each decay process are presented in Tab.1 and it show that the values of $f_{(gg)}(r)$ changes slowly when r goes from 0.275 to 0.381. The Feynman diagrams in Fig.[1] indicate the process $\Upsilon \rightarrow J/\psi + gg$ can be viewed as $\Upsilon \rightarrow gg^{(*)}g^{(*)b}$ followed by $g^{(*)}g^{(*)} \rightarrow J/\psi + g$. Then the normalized Γ_{Nor}^g can cancel part of the contribution at m_b scale, so similar to what is done in Ref.[21] we choose the scale of α_s to be $2m_c$. The theoretical uncertainties of $\Upsilon \rightarrow J/\psi + ggg$ can be analyzed in the same way.

By setting the default parameter choice: $m_b = 4.73\text{GeV}$, $r = 0.327$, $\langle \mathcal{O}_1^\psi(^3S_1) \rangle = 1.25\text{GeV}^3$ and

^a The contribution of $\Upsilon \rightarrow \gamma^* \rightarrow q\bar{q}$ is excluded.

^b $g^{(*)}$ means the gluon can either be virtual or real.

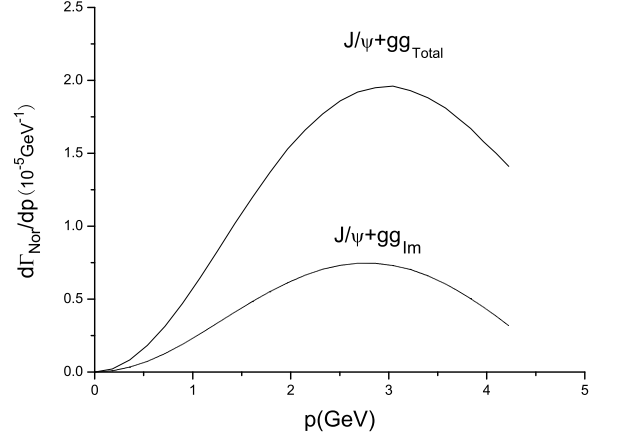


FIG. 3: The normalized partial width of the $\mathcal{O}(\alpha_s^6)$ $\Upsilon \rightarrow J/\psi + gg$ process as function of J/ψ momentum $p_{J/\psi}$. The solid line is the total result, and the dashed line is the contribution from the imaginary part in the Feynman amplitude.

$\alpha_s(2m_c) = 0.26$, we obtain

$$\mathcal{B}^g = \Gamma_{\text{Nor}}^g \times \mathcal{B}(\Upsilon \rightarrow ggg) = 0.47 \times 10^{-4}. \quad (13)$$

Using the same inputs to re-estimated the result in Ref.[27] and adding up it with the contribution in Eq.[13], we obtain the total CS singlet prediction

$$\mathcal{B}(\Upsilon \rightarrow J/\psi + X) = 0.90 \times 10^{-4}, \quad (14)$$

where the total contribution from the $\mathcal{O}(\alpha_s^6)$ $J/\psi + gg$ and $J/\psi + ggg$ processes is as important as those calculated in Ref.[27]. It is clear that the uncertainties are from the b quark mass m_b , the scaleless functions $f_{gg}(r)$ and $f_{4g}(r)$, and the choice of the scale of α_s . To estimate the uncertainty, we used $m_b = 4.6\text{GeV}$, $r = 0.296$ and $\mu = 2m_c$ for upper bound; $m_b = 4.9\text{GeV}$, $r = 0.361$ and $\mu = 2m_c$ for lower bound, then the branching ratio is represented as:

$$\mathcal{B}(\Upsilon \rightarrow J/\psi + X) = 0.90_{-0.31}^{+0.49} \times 10^{-4}. \quad (15)$$

Furthermore the total branching ratio turns to be a much smaller value 6.3×10^{-5} (5.2×10^{-5}) by choosing the scale to be $2m_b$ ($2\sqrt{m_b m_c}$) and $\alpha_s(2m_b) = 0.18$ ($\alpha_s(2\sqrt{m_c m_b}) = 0.21$) and keeping the other parameters the same as for the central value of the branching ratio.

The experimental result in Eq.(1) includes the feed-down contributions of χ_{cJ} , which are $< 8.2, 11, 10$ percents for $J = 0, 1, 2$ respectively, and 24% feed-down contribution of $\psi(2S)$. Removing the feed-down contributions, the branching ratio of direct J/ψ production in Υ decay would be

$$\mathcal{B}(\Upsilon \rightarrow J/\psi_{\text{direct}} + X) = 3.52 \times 10^{-4}. \quad (16)$$

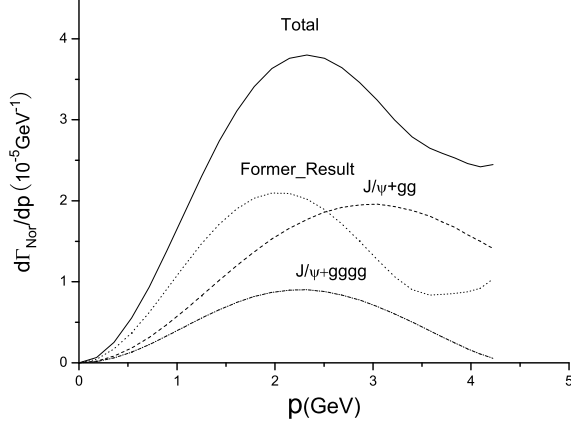


FIG. 4: The normalized partial width for the CS J/ψ production in Υ decay as function of J/ψ momentum $p_{J/\psi}$. The solid line is the total result. the dashed line is the contribution of the $\mathcal{O}(\alpha_s^6)$ $\Upsilon \rightarrow J/\psi + gg$ process. The dashed-dot line is the $100\times$ the contribution $\mathcal{O}(\alpha_s^6)$ $\Upsilon \rightarrow J/\psi + gggg$. The dot line is the contribution calculated in Ref.[27], which includes the $\mathcal{O}(\alpha_s^5)$ $\Upsilon \rightarrow J/\psi + c\bar{c}g$ and $\mathcal{O}(\alpha^2\alpha_s^2)$ $\Upsilon \rightarrow J/\psi + gg$ and $\Upsilon \rightarrow J/\psi + c\bar{c}$ processes.

Which is about 3.8 times larger than the current CS prediction.

For the J/ψ momentum spectrum, the normalized decay widths defined in Eq.(12) are used to present the results with default parameter choice. Both the total result (solid line) and the imaginary part contribution (dashed line) for $\mathcal{O}(\alpha_s^6)$ $J/\psi + gg$ process are shown in Fig.[3] with very similar shape. A summarized CS contribution to the $p_{J/\psi}$ distribution of the normalized decay width is shown in Fig.[4]. And we find the peak of total result curve is at $p_{J/\psi} = 2.7\text{GeV}$, which is a little larger than that of the CLEO measurement[31].

It is found that the J/ψ production in association with $c\bar{c}$ pair is an important mechanism for J/ψ electroproduction[2, 3] in the Belle experiment. And theoretically, the contribution of $p\bar{p} \rightarrow J/\psi + c\bar{c} + X$ process to J/ψ hadroproduction at the Tevatron is also found to be non-ignorable[35, 36]. The ratio of J/ψ production in association with $c\bar{c}$ pair to J/ψ plus anything may also be a good probe to reveal the J/ψ production mechanism in Υ decay and to clarify the conflict between the CLEO measurement and theoretical prediction. Choosing $\alpha_s(2m_c) = 0.259$, we give the CS prediction for the ratio R_{cc}

$$R_{cc} = \frac{\mathcal{B}(\Upsilon \rightarrow J/\psi + c\bar{c} + X)}{\mathcal{B}(\Upsilon \rightarrow J/\psi + X)} = 0.39^{+0.21}_{-0.20}, \quad (17)$$

where the center, upper and lower bound values correspond to $r = 0.327, 0.296$ and 0.361 respectively, and the associated charmed particles process includes the $\mathcal{O}(\alpha_s^5)$ $\Upsilon \rightarrow J/\psi + c\bar{c}g$ sub-process, which is dominant, and

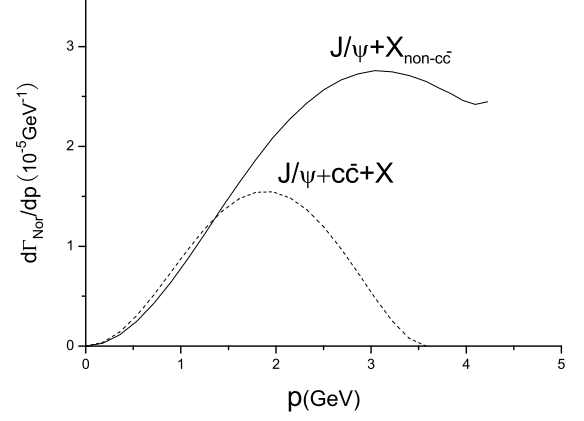


FIG. 5: The normalized partial width of Υ decay into $J/\psi + c\bar{c} + X$ process as function of J/ψ momentum $p_{J/\psi}$ (dashed line) and that for $J/\psi + X$ production (solid line).

$\mathcal{O}(\alpha^2\alpha_s^2)$ $\Upsilon \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ sub-process. On the contrary, the CO prediction of R_{cc} is only at the level of 1%[21], which is quite different with the CS prediction. Unlike the branching ratio, the theoretical prediction of R_{cc} only depends on r and α_s , which results in a relatively small uncertainty. Particularly, if we drop the contribution of QED part, R_{cc} is just proportional to α_s . In Ref.[4, 5], the authors find the enhancement of the NLO QCD corrections is large in $e^+e^- \rightarrow J/\psi + c\bar{c}$ process. It indicates that the result in Eq.[17] is only a very preliminary result and to get a more solid predictions the contribution of the NLO QCD corrections to $\Upsilon \rightarrow J/\psi + c\bar{c} + g$ process should be taken into account. Calculating the NLO QCD corrections to $\Upsilon \rightarrow J/\psi + c\bar{c} + g$ process is beyond the scope of this work and will not be discussed here. In the end, the J/ψ momentum spectra for the associated process and non- $c\bar{c}$ process are given in Fig.[5] for comparison.

In summary, in this work, we calculate the $\mathcal{O}(\alpha_s^6)$ CS contribution of $\Upsilon \rightarrow J/\psi + gg$ and $\Upsilon \rightarrow J/\psi + gggg$ processes to the inclusive J/ψ production in Υ decay. The branching ratio is estimated in two ways. In the first way, the numerical results of partial width and branching ratios are all evaluated directly. And we find its result is more close to the experimental data when the values of the parameters are properly chosen, which is also coincident with a rough estimated result mentioned in Ref.[24]. However, the uncertainty of this way is very large and the branching can be in a wide range of $2.3 \times 10^{-5} \sim 1.7 \times 10^{-4}$. In the second way, the branching ratio is calculated by using the normalized decay width, which seems more reliable. After combining the present result with the contribution calculated in our previous work[27], we find now the total CS prediction is about 0.92×10^{-4} , which is still about 3.8 times less than the experimental value 3.2×10^{-4} for direct J/ψ production

given by the CLEO Collaboration[31]. We also calculate the J/ψ momentum spectrum and find the peak of the color-singlet curves is close to that of the CLEO result, although being a little larger. Besides the the branching ratio and the J/ψ spectrum, we also study the ratio $R_{cc} = \mathcal{B}(\Upsilon \rightarrow J/\psi + c\bar{c} + X)/\mathcal{B}(\Upsilon \rightarrow J/\psi + X)$, and find the CS prediction is much larger than that of the CO. Since the associated charmed meson process can be measured separately, a analysis of the ratio is expected to perform in the CLEO, Babar or Belle experiments. Now there is still large discrepancy between the CLEO results and NRQCD predictions. There are two points to be addressed: first the J/ψ production mechanism is not well understood yet, and the existence of the CO mechanism is still under debate; second the higher QCD corrections are not included completely. Therefore, to understand

the J/ψ production mechanism in Υ decay and moreover in $p\bar{p}$ collisions at the Tevatron. Further theoretical and experimental work are necessary.

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